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Faddeev–Jackiw Quantization Method in Conformal Three-Dimensional Supergravity

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The conformal supergravity in three space-time dimensions is described by a pure Lorentz–Chern–Simons term. This system has constraints on curvatures and so it is a higher-derivative gauge model. The dynamical properties of this model are analyzed by means of the supersymmetric extension of the Faddeev–Jackiw symplectic quantization method. Using this algorithm in the first-order formalism, we study the gauge supersymmetric transformations and we find the constraints of the model.

1. INTRODUCTION

As is well known, pure gravity in three space-time dimensions is described by a Chern–Simons gauge theory of the Poincaré group or the de Sitter group (Uematsu, 1985; Achucarro and Townsend, 1986; Witten, 1988; Koehler *et al.*, 1990; Grignani and Nardelli, 1991; Campbell *et al.*, 1990). The extended Poincaré supergravities in three space-time dimensions such as Chern–Simons gauge theories are obtained by performing an Inonü–Wigner contraction of the Lie superalgebra associated to the de Sitter or the anti-de Sitter groups, namely *Osp*(1/2; *C*) and *Osp*(1/2; */R*) × *Osp*(1/2; *R*), respectively (Koehler *et al.*, 1991a,b). Due to its important role in supergravity, the supersymmetric extension of the gravitational Lorentz–Chern–Simons term has been treated by several authors. For instance, van Nieuwenhuizen (1985), by gauging the superconformal algebra in three space-time dimensions associated to the group *Osp*(1/4), found the general properties of the model. In that paper the suitable three-dimensional action is constructed starting from the

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Pontryagin invariant in four dimensions. There are several reasons to study conformal supergravity models in three dimensions. Among them an important fact from the quantization point of view is that in conformal supergravity the gauge algebra closes off shell. Moreover, when conformal supergravity models are considered it is possible to show that local supersymmetry can exist even in flat space-time.

Like gauge theories, the conformal supergravity model in three dimensions is a constrained system and the requirement of invariance under local symmetries naturally implies constraints on curvatures. Consequently, when the model is treated in the second-order formalism, the higher-derivative character of the theory is made evident. This is an additional difficulty of the model. The Dirac formalism (Dirac, 1964; Sundermeyer, 1982) is the usual way to deal with constrained systems and has long been used in a large number of systems both in quantum mechanics as well as in field theory. However, in complicated systems the computation of the basic structure, i.e., the Dirac brackets, is heavy and tedious (Foussats *et al.*, 1992).

An alternative way to treat constrained systems was proposed by Faddeev and Jackiw (1988) (FJ) and in some cases is more economical than the Dirac method. This happens because in the FJ construction there is in general a smaller number of constraints (Kulshreshtha and Muller-Kirsten, 1991; Barcelos-Neto and Wotzasek, 1992a,b, 1993; Montani and Wotzasek, 1993; Horta-Barreira and Wotzasek, 1992). The FJ symplectic quantization method is based on first-order Lagrangians. This is not a serious restriction because any system can be written in a first-order formalism by enlarging the configuration space introducing proper auxiliary fields. As can be shown, the generalized brackets obtained from the equation of motion are equal to those obtained by means of the Dirac formalism, producing the same dynamical results (Costa and Girotti, 1988; Govaerts, 1990). In the FJ symplectic formalism the classification of constrained or unconstrained systems is related to the singular or nonsingular behavior of the fundamental symplectic two-form. Moreover, the classification of constraints into first class, second class, and so on has no meaning and there are only constraints associated with gauge symmetries. So in this method there are in general fewer constraints than generated by the Dirac algorithm. Hence, we can expect that the algebraic manipulations needed in the treatment of constrained systems could be shortened.

The FJ symplectic formalism has been well studied (Barcelos-Neto and Srivastava, 1991; Barcelos-Neto and Wotzasek, 1992a,b; Montani and Wotzasek, 1993) and a supersymmetric extension of this method to include Grassmann dynamical variables can be found in Govaerts (1990), but it has not often been used in supersymmetric systems and even less in supergravity models.

In this paper we will use the FJ symplectic quantization formalism to describe the first-order formalism of conformal supergravity in three space-time dimensions. In Section 2 we briefly introduce the main dynamical features of the model. In Section 3 we summarize the principal characteristics of the supersymmetric extension of the FJ symplectic quantization formalism. Finally, in Section 4 we apply the method to the supersymmetric model under consideration.

2. CONFORMAL SUPERGRAVITY IN THE FIRST-ORDER FORMALISM. DEFINITIONS AND PRELIMINARIES

Conformal supergravity in three space-time dimensions is the gauge theory obtained by gauging the superconformal algebra associated to the group $Osp(1/4)$ (van Nieuwenhuizen, 1985). The 14 generators are: the Lorentz generators M_{ab} $(a, b = 1, 2, 3)$, the dilaton generator D, and the translation and boost generators P_a and K_a, respectively. Moreover, the odd generators are the ordinary and conformal supersymmetry generators Q^α and S^α ($\alpha = 1, 2$), respectively. The (anti) commutation relation of the conformal superalgebra, the notation, and conventions we use are those given in (van Nieuwenhuizen, 1985; Foussats *et al.*, 1992).

The action of the conformal supergravity in three space-time dimensions is obtained from the four-dimensional Pontryagin invariant by peeling off an exterior derivative. The Lagrangian density remains defined by means of the three-form

$$\mathcal{L} = \gamma_{AB} R^B \wedge \mu^A + \frac{1}{6} C_{ABC} \mu^C \wedge \mu^B \wedge \mu^A \tag{2.1}$$

where the two-form curvature R^B is defined by

$$R^B = d\mu^B - \frac{1}{2} C^B_{CD} \mu^D \wedge \mu^C \tag{2.2}$$

The one-form gauge fields for the different values of the compound index A are

$$\mu^A = (\omega^{ab}, V^a, f^a, b, \xi, \varphi) \tag{2.3}$$

where ω^{ab} is the spin connection, V^a the dreibein, f^a the proper conformal boost gauge field, b the dilaton, and ξ (Q-gravitino) and φ (S-conformal gravitino) are two Majorana spinors. The totally antisymmetric graded structure constants C_{ABC} are related to the graded structure constants C^B_{CD} through the constant symmetric Killing metric γ_{AB} of $Osp(1/4)$ by the equation

$$C_{ABC} = \gamma_{AD} C^D_{BC} \tag{2.4}$$

The explicit expressions for the six curvatures defined in (2.2) are

$$R^{ab}(\omega) = \mathcal{R}^{ab}(\omega) - 2(V^a \wedge f^b - V^b \wedge f^a) + 2\bar{\xi} \wedge \tau^{ab}\varphi \quad (2.5a)$$

$$R^a(V) = \mathbf{D}V^a - \bar{\xi} \wedge \tau^a\xi \quad (2.5b)$$

$$R^a(f) = \mathbf{D}f^a + \bar{\varphi}\tau^a\varphi \quad (2.5c)$$

$$R(b) = db + 2V^a \wedge f_a - 2\bar{\xi} \wedge \varphi \quad (2.5d)$$

$$\rho(\xi) = \mathbf{D}\xi + V_a \wedge \tau^a\varphi \quad (2.5e)$$

$$\rho(\varphi) = \mathbf{D}\varphi - f_a \wedge \tau^a\xi \quad (2.5f)$$

where

$$\mathcal{R}^{ab}(\omega) = d\omega^{ab} + \omega^{ac} \wedge \omega_c^b \quad (2.6a)$$

$$\mathbf{D}V^a = (\mathcal{D} - b) \wedge V^a, \qquad \mathbf{D}f^a = (\mathcal{D} + b) \wedge f^a \quad (2.6b)$$

$$\mathbf{D}\xi = \left(\mathcal{D} - \frac{1}{2}b\right) \wedge \xi, \qquad \mathbf{D}\varphi = \left(\mathcal{D} + \frac{1}{2}b\right) \wedge \varphi \quad (2.6c)$$

and the Lorentz-covariant derivatives of V^a, f^a, ξ, and φ are, respectively,

$$\mathcal{D}V^a = dV^a + \omega^{ab} \wedge V_b \qquad \text{idem for } f^a \quad (2.7a)$$

$$\mathcal{D}\xi = d\xi + \frac{1}{4}\omega^{ab} \wedge \tau_{ab}\xi \qquad \text{idem for } \varphi \quad (2.7b)$$

To treat this supersymmetric model in the framework of the FJ symplectic formalism it is necessary to write the Lagrangian density (2.1) in components

$$\mathcal{L} = (-g)^{1/2}\epsilon^{\sigma\nu\rho}\left[\gamma_{AB}R^B_{\sigma\nu}\mu^A_\rho + \frac{1}{6}C_{ABC}\mu^C_\sigma\mu^B_\nu\mu^A_\rho\right], \qquad \sigma, \nu, \rho = 0, 1, 2 \quad (2.8)$$

and next it must be written in the symplectic form by writing the time derivative explicitly.

3. SUPERSYMMETRIC EXTENSION OF THE FADDEEV–JACKIW QUANTIZATION METHOD

Before treating the model described in the above section, we review the main results of the symplectic formalism when used in supersymmetric field-theoretic models.

The FJ symplectic quantization method is based on an action containing only first-order time derivatives. The most general first-order action contains a Lagrangian density specified in terms of two arbitrary functionals $K_A(q^A)$ and $V(q^A)$ which is given by

$$L(q_A, \dot{q}^A) = \dot{q}^A K_A(q^A) - \mathbf{V}(q^A) \tag{3.1}$$

The functionals $K_A(q^A)$ are the components of the canonical one-form $K(q) = K_A(q)\, dq^A$ and the functional $\mathbf{V}(q)$ is the symplectic potential. Both are of even Grassmann parity and therefore $K_A(q)$ has Grassmann parity $|A|$, where the general compound index A runs over the different ranges of the complete set of variables. The set of field dynamical variables q^A is given by the original set of fields plus a set of auxiliary fields necessary to bring the system into its first-order form (3.1) and this set defines the extended configuration space.

The Euler-Lagrange equations of motion obtained from (3.1) are

$$\sum_B (-1)^{|B|} M_{AB} \dot{q}^B - \frac{\partial \mathbf{V}}{\partial q^A} = 0 \tag{3.2}$$

The elements of the simplectic supermatrix $M_{AB}(q)$ are the components of the symplectic two-form $M(q) = dK(q)$. The exterior derivative of the canonical one-form $K(q)$ is written as the generalized curl constructed with functional derivatives and so the components are given by

$$M_{AB}(x, y) = \frac{\delta K_B(y)}{\delta q^A(x)} - (-1)^{|A||B|} \frac{\delta K_A(x)}{\delta q^B(y)} \tag{3.3}$$

By definition, the Grassmann parity of the supermatrix M_{AB} is $(|A| + |B|)$ and the symmetry property is

$$M_{AB} = -(-1)^{|A||B|} M_{BA} \tag{3.4}$$

When the simplectic supermatrix M_{AB} is nonsingular, it defines the symplectic two-form characterizing the dynamical system described by (3.1). From the equations of motion, (3.2) is

$$\dot{q}^B = (M^{AB})^{-1} \frac{\partial \mathbf{V}}{\partial q^A} \tag{3.5}$$

As the symplectic potential is just the Hamiltonian of the system, equation (3.5) is written

$$\dot{q}^B = [\mathbf{V}, q^B\} = \frac{\partial \mathbf{V}}{\partial q^A} [q^A, q^B\} \tag{3.6}$$

where $[q^A, q^B\} = (M^{AB})^{-1}$ are the generalized graded brackets of the FJ symplectic formalism. As is known, the elements $(M^{AB})^{-1}$ of the inverse of the symplectic supermatrix M_{AB} correspond to the graded Dirac brackets of the theory. Transition to the quantum theory is realized as usual by replacing classical fields by quantum field operators acting on some Hilbert space. Therefore, in this case the FJ and the Dirac methods are equivalent.

On the other hand, in gauge-invariant field theories, besides the true dynamical degrees of freedom there are also gauge degrees of freedom, and so first-class constraints exist and the supermatrix M_{AB} is singular. In the FJ formalism the constraints appear as algebraic relations and they are necessary to maintain the consistency of the field equations of motion. In such a case there exist n ($n < N$) left (or right) zero modes $\mathbf{v}_{(k)}$ ($k = 1, \ldots, n$) of the supermatrix M_{AB}, where each $\mathbf{v}_{(k)}$ is a column vector with N entries $v^A_{(k)}$. So the zero modes satisfy the equation

$$\sum_A v^A_{(k)} M_{AB} = 0 \tag{3.7}$$

where $A, B = 1, \ldots, N$. That is the case of the dynamical system described by the Lagrangian density (2.8).

Consequently, from the equations of motion (3.2) we can write

$$\Omega_{(k)} = \int dx\, v^A_{(k)}(x, t) \frac{\delta}{\delta q^A(x, t)} \int dy\, \mathbf{V}(y, t) = 0 \tag{3.8}$$

The quantities $\Omega_{(k)}$ (of Grassmann parity $|k|$) are the constraints in the FJ symplectic formalism, and they are introduced in the Lagrangian by using suitable Lagrange multipliers $\Lambda^{(k)}$ of Grassmann parity $|k|$:

$$L = \dot{q}^A K_A(q) - \Lambda^{(k)}\Omega_{(k)} - \mathbf{V}(q) \tag{3.9}$$

In equation (3.9) we have assumed that $q^A(x)$ represents any field belonging to the symplectic set and the compound index A runs over the range $1, \ldots, N - n$. Therefore, the submatrix $\overline{M}_{AB}$ of the supermatrix (3.3) is nonsingular. At this point one can run the symplectic algorithm once again, enlarging the configuration space by considering the set of variables $(q^A, \xi^{(k)})$. This is done by redefining the $\Lambda^{(k)}$ variables as

$$\Lambda^{(k)} = -\dot{\xi}^{(k)} \tag{3.10}$$

Therefore, the first-iterated Lagrangian is written

$$L^{(1)} = \dot{q}^A K_A(q) + \dot{\xi}^{(k)}\Omega_{(k)} - \mathbf{V}^{(1)}(q) \tag{3.11}$$

where $\mathbf{V}^{(1)}(q) = \mathbf{V}(q)|_{\Omega_{(k)}=0}$.

In terms of the new set of dynamical variables, the symplectic supermatrix in compact notation is written

$$M^{(1)} = \begin{pmatrix} \overline{M}_{AB} & \delta\Omega_{(k)}/\delta q^B \\ -(-1)^{|k||B|}(\delta\Omega_{(k)}/\delta q^B)^T & 0 \end{pmatrix} \tag{3.12}$$

where $\overline{M}_{AB}$ (submatrix of M_{AB}) represents the square nonsingular matrix constructed from the original symplectic set of field variables. The notation $\delta\Omega_{(k)}/\delta q^B$ represents a rectangular supermatrix.

This iterative procedure modifies the symplectic supermatrix until all the nonorthogonal zero modes have been eliminated. That means that the algorithm must be repeated until no new constraint is generated. As we will see, for gauge-invariant theories, the algorithm is not able to generate an invertible symplectic supermatrix. Therefore, to obtain the generalized brackets, gauge-fixing conditions must be imposed.

In the next section we apply the above method to three-dimensional conformal supergravity in the first-order formalism.

4. THE FADDEEV–JACKIW METHOD IN THREE-DIMENSIONAL CONFORMAL SUPERGRAVITY. THE CONSTRAINTS IN THE FIRST-ORDER FORMALISM

The Lagrangian density (2.7) can be easily written in the form (3.1),

$$\mathcal{L} = (g^{1/2}\epsilon^{ij}\gamma_{AB}\mu_j^A)\dot{\mu}_i^B - \mathbf{V}(\mu_\nu^A) \tag{4.1}$$

The initial set of symplectic variables defining the extended configuration space is $\mu_\nu^A = (\mu_i^A, \mu_0^A)$, where μ^A are defined in equation (2.3). From (4.1) we note that the variables μ_0^A do not appear in the symplectic part of the Lagrangian.

The matrix elements of the symplectic supermatrix can be easily obtained and they read

$$\overline{M}_{\mu_i^A(x),\mu_j^B(y)} = -(-1)^{|A||B|}2g^{1/2}\epsilon^{ij}\gamma_{AB}\delta(x-y) \tag{4.2}$$

which corresponds to the square nonsingular supermatrix constructed from the original symplectic set of variables μ_i^A, and

$$M_{\mu_0^A(x),\mu_i^B(y)} = M_{\mu_0^A(x),\mu_0^B(y)} = 0 \tag{4.3}$$

Consequently, the supermatrix

$$M^{(0)} = \begin{pmatrix} \overline{M}_{\mu_i^A,\mu_j^B} & 0 \\ 0 & 0 \end{pmatrix} \tag{4.4}$$

is obviously singular. This supermatrix has zero eigenvalues whose corresponding zero modes we call v_0^A and which satisfy the equation (3.8).

On the other hand in equation (4.1) the symplectic potential $\mathbf{V}(\mu^A)$ is given by

$$\mathbf{V}(\mu^A) = -2g^{1/2}\epsilon^{ij}\gamma_{AB}\mu_0^A(\partial_i\mu_j^B - \tfrac{1}{2}C_{CD}^B\mu_i^D\mu_j^C) \tag{4.5}$$

Therefore, in this case equation (3.8) is written

$$\int dx\, v_{\mu_0^A}(x, t) \int dy\, \frac{\delta \mathbf{V}(y, t)}{\delta\mu_0^A(x, t)}$$

$$= \int dx\, v_{\mu_0^A}(x, t)(-2g^{1/2}\gamma_{AB}\epsilon^{ij}R_{ij}^B) = 0 \tag{4.6}$$

Since equation (4.6) is satisfied for arbitrary zero-mode components $v_{\mu_0^A}(x, t)$, it follows that

$$\Omega_A = -2g^{1/2}\epsilon^{ij}\gamma_{AB}R_{ij}^B = 0 \tag{4.7}$$

Now, the constraints (4.7) are introduced in the Lagrangian by means of Lagrange multipliers and the first-iterated Lagrangian takes the form

$$\mathcal{L}^{(1)} = (g^{1/2}\epsilon^{ij}\gamma_{AB}\mu_j^A)\dot{\mu}_i^B + \Omega_A\dot{\lambda}^A - \mathbf{V}^{(1)}(\mu) \tag{4.8}$$

where

$$\mathbf{V}^{(1)}(\mu) = \mathbf{V}\,|_{\Omega_A=0} = 0 \tag{4.9}$$

The new set of variables is (μ_k^A, λ^A), and the new singular once-iterated supermatrix has the following matrix elements:

$$M^{(1)}_{\mu_i^A(x),\mu_j^B(y)} = \overline{M}_{\mu_i^A(x),\mu_j^B(y)} = -(-1)^{|A||B|}2g^{1/2}\epsilon^{ij}\gamma_{AB}\delta(x - y) \tag{4.10a}$$

$$M^{(1)}_{\mu_i^A(x),\lambda^B(y)} = -(-1)^{|A||B|}M^{(1)}_{\lambda^B(y),\mu_i^A(x)} = \frac{\delta\Omega_A(y)}{\delta\mu_i^B(x)} \tag{4.10b}$$

$$M_{\lambda_A(x),\lambda_B(y)} = 0 \tag{4.10c}$$

From equations (4.10a)–(4.10c) we can conclude that the first-iterated supermatrix $M^{(1)}_{AB}$ is also singular. Due to the symplectic potential $\mathbf{V}^{(1)}(\mu) = 0$, by repeating once the procedure, no new constraints are obtained. Therefore, the constraints Ω_A cannot be eliminated because they are the true first-class constraints related to the gauge symmetries of the model. Moreover, as is well known, the zero modes associated to the singular supermatrix $M^{(1)}_{AB}$ generate a symmetry on the constraint surface (Montani and Wotzasek, 1993). It is possible to show that the components of these zero modes are the correct gauge supersymmetry generators for the spatial components of the gauge fields. In fact we have that

$$\delta_\epsilon \mu_i^A = v_{\mu_i^A}^B \epsilon_B = -\overline{M}_{\mu_i^A(x),\mu_j^C(y)} \frac{\delta \Omega^B}{\delta \mu_j^C} \epsilon_B \tag{4.11}$$

where ϵ_B are the infinitesimal parameters corresponding to the supersymmetric transformations. After some algebra we find

$$\delta_\epsilon \mu_i^A = \int dy\, [\partial_i \epsilon^A(y) + C_{BC}^A \mu_i^B(y) \epsilon^C(y)] \delta(x - y) \tag{4.12}$$

which can be written

$$\delta \mu_i^A = (\nabla_i \epsilon)^A \tag{4.13}$$

Now, by making the identification $\dot{\lambda}_A = -\mu_0^A$ and carrying out the gauge transformation on the Lagrange multiplier μ_0^A, we similarly find

$$\delta \mu_0^A = (\nabla_0 \epsilon)^A \tag{4.14}$$

Therefore, the infinitesimal transformation (4.13) on the constraint surface can be extended to the full space-time by writing

$$\delta \mu_\nu^A = (\nabla_\nu \epsilon)^A \tag{4.15}$$

This last equation is the usual form of a gauge transformation on the gauge fields.

As noted above, when we deal with gauge-invariant theories, once the iterative FJ procedure is finished the simplectic supermatrix is even singular. The singular symplectic supermatrix contains the complete information about all the supersymmetries present in the model.

With the aim of obtaining a nonsingular supermatrix, gauge-fixing conditions must be added to the classical Lagrangian. The gauge-fixing terms break the gauge symmetries in the symplectic potential, giving rise to an invertible supermatrix and providing the generalized FJ graded brackets which correspond to the graded Dirac brackets of the model. Therefore, once the nonsingular supermatrix is found, the complete canonical information about the dynamical system is obtained.

5. CONCLUSIONS

In summary, we have found the constraints of the conformal supergravity model in the framework of the FJ simplectic method. In this context, the gauge supersymmetries in the first-order formalism were also analyzed. It is clear that the zero modes of the symplectic supermatrix constructed by this method are closely related to the generators of gauge supersymmetries. In this context the unique constraints are those associated to gauge supersymmetries, and so the role of generators of the gauge supersymmetries assigned to these first-class constraints is clear. At least in the first-order formalism, the algebraic manipulations to find the constraints are shortened in comparison with the Dirac procedure.

On the other hand, it is convenient to give the gauge-fixing conditions in the second-order formalism (van Nieuwenhuizen, 1981) and not in the first-order one. As usual, the second-order formalism is obtained by considering the two constraint equations on curvatures (2.5b) and (2.5e) as strongly equal to zero. This allows us to eliminate the spin connection and the conformal gravitino as dynamical fields. The final Lagrangian density only contains as dynamical fields the graviton (the dreibein V^a) and the Q-gravitino. The second-order formalism makes evident the higher-derivative character of the model. Second time derivatives appear on the graviton field that cannot be eliminated, and so the use of the Dirac formalism in this model is very complicated (Foussats *et al.*, 1992).

In the framework of the FJ it is necessary to introduce a set of auxiliary variables to rewrite the Lagrangian density of the system into its first-order form (3.1). We have no answer yet about the usefulness of applying the FJ method to this kind of complicated system. The problem of treating higher-derivative models by using the symplectic method will be the subject of a future paper.

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